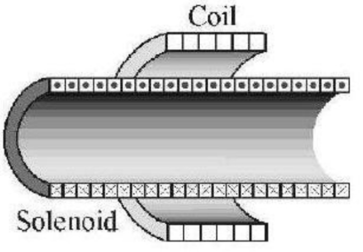
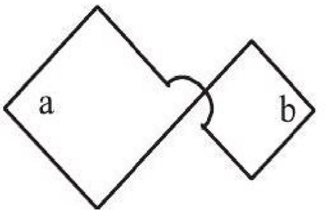
Electromagnetic Induction

1. Two concentric coplanar circular loops made of wire, with resistance per unit length $10\Omega/\mathrm{m}$ have diameters 0.2 m and 2 m. A time varying potential difference $(4+2.5t)$ volt is applied to the larger loop. Calculate the current (in A) in the smaller loop.
2. In figure a 120-turn coil of radius 1.8 cm and resistance 5.3Ω is placed outside a solenoid. The current in the solenoid is 1.5 A and it reduces to zero at a steady rate in 25 ms. What current (in mA) appears in the coil? The number of turns per unit length of the solenoid is 220 turns /cm and its diameter $D=3.2$ cm.



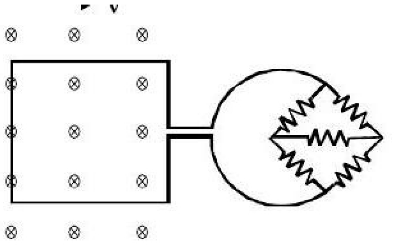
3. The current in a coil of self-induction 2.0 henry is increasing according to $i=2\sin\ t^2$ ampere. Find the amount of energy (in joule) spent during the period when the current changes from 0 to 2 ampere.
4. The current in an RL circuits drops from 1.0 A to 10 mA in the first second following removal of the battery from the circuit. If L is 10 H, find the resistance R (in ohm) in the circuit.
5. An air plane, with a 20 m wing spread is plying at $250\ \mathrm{m/s}$ straight south parallel to earth's surface. The earth's magnetic field has a horizontal component of $2\times10^{-5}\ \mathrm{Wb/m^2}$ and angle of dip is 60°. Calculate the induced emf (in V) between the plane tips.
6. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1 s, the change in the energy (in joule) of the inductance is:
7. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of $5.0\ \mathrm{ms^{-1}}$, at right angles to the horizontal component of the earth's magnetic field, of $0.3\times10^{-4}\ \mathrm{Wb/m^2}$. The value of the induced emf (in V) in wire is :
8. A uniform magnetic field B exists in a direction perpendicular to the plane of a square frame made of copper wire. The wire has a diameter of 2 mm and a total length of 40 cm. The magnetic field changes with time at a steady rate $\frac{dB}{dt}=0.02\frac{T}{s}$. Find the current (in A) induced in the frame. Resistivity of copper $=1.7\times10^{-8}$ W-m.
9. A conducting circular loop having a radius of 5.0 cm, is placed perpendicular to a magnetic field of 0.50 T. It is removed from the field in 0.50 s. Find the average emf (in V) produced in the loop during this time.
10. A coil of inductance 1 H and resistance 10Ω is connected to a resistanceless battery of emf 50 V at time $t=0$. Calculate the ratio of the rate at which magnetic energy is stored in the coil to the rate at which energy is supplied by the battery at $t=0.1$ s.
11. A long solenoid having 200 turns per cm carries a current of 1.5 amp. At the centre of it is placed a coil of 100 turns of cross-sectional area $3.14\times10^{-4}\ \mathrm{m^2}$ having its axis parallel to the field produced by the solenoid. When the direction of current in the solenoid is reversed within 0.05 sec, the induced e.m.f. (in V) in the coil is
12. A circular disc of radius 0.2 meter is placed in a uniform magnetic field of induction $\frac{1}{\mathrm{p}}(\mathrm{Wb/m^2})$ in such a way that its axis makes an angle of 60° with $\vec{B}$. The magnetic flux (in Wb) linked with the disc is:

13. A plane loop, shaped as two squares of sides $a = 1$ m and $b = 0.4$ m is introduced into a uniform magnetic field perpendicular to the plane of loop. The magnetic field varies as $B = 10^{-3}\sin$



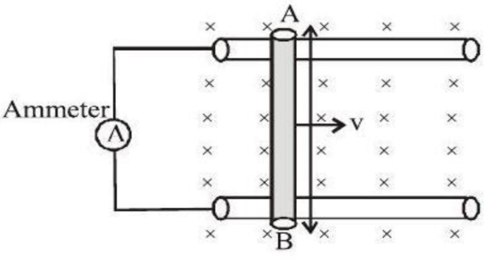
(100t) T. The amplitude of the current (in A) induced in the loop if its resistance per unit length is $r = 5\ m\Omega\ m^{-1}$ is

14. A square metal loop of side 10cm and resistance 1Ω is moved ∘ $\vec{\otimes}_{\otimes}$ v ∘ with a constant velocity partly inside a uniform magnetic field ∗ of $2Wbm^{-2}$, directed into the paper, as shown in the figure.



The loop is connected to a network of five resistors each of value 3Ω. If a steady current of 1 mA flows in the loop, then the speed (in cms^{-1}) of the loop is

15. If the rod is moving with a constant velocity of 12 cm/s then the power (in watt) that must be supplied by an external force in maintaining the speed will be



(Given $B = 0.5$ Tesla, $l = 15$ cm, $v = 12$ cm/s, Resistance of rod $R_{AB} = 9.0\ m\Omega$)

SOLUTIONS

1. **(1.25)** The resistance of the loops,

$$R_1 = 2\pi r_1 \times 10 = 2\pi \times 0.1 \times 10$$
$$= 6.28\,\Omega.$$

and $$R_2 = 2\pi r_2 = 2\pi \times 1 \times 10$$
$$= 62.8\,\Omega.$$

Flux in the smaller loop, $\phi = B_2 A_1$

$$= \frac{\mu_0 i_2}{2r_2}\pi r_1^2$$

$$= \frac{\mu_0\left[\frac{V}{R_2}\right]\pi r_1^2}{2r_2}$$

$$= \frac{\mu_0\frac{[4+2.5t]}{R_2}\pi r_1^2}{2r_2}$$

The induced current, $$i_1 = \frac{e}{R_1} = \frac{[d\phi/dt]}{R_1}$$

After substituting the value and simplifying we get

$$i = 1.25\,\text{A}.$$

2. **(30)** The induced emf, $$|e| = A\left(\frac{\Delta B}{\Delta t}\right)$$

$$= (\pi R)^2 \mu_0 n\left(\frac{\Delta i}{\Delta t}\right)$$

$$= \pi(1.8\times10^{-2})^2 \times (4\pi\times10^{-7}) \times 220 \times 10^2$$

$$\times\left(\frac{1.5}{25\times10^{-3}}\right) = 68.6\,\text{mV}$$

Current, $$i = \frac{e}{R} = 30\text{ mA}$$

3. **(4)** For $$0 = 2\sin t^2 \Rightarrow t = 0$$

and $$2 = 2\sin^2 t \Rightarrow t^2 = \frac{\pi}{2},$$

$\therefore$ $$t = \sqrt{\frac{\pi}{2}}$$

The energy spent, $$E = \frac{1}{2}Li^2$$

$$= \frac{1}{2}L(2\sin t^2)^2$$
$$= 2L\sin^2 t$$
$$= 2\times 2\sin^2 \pi/2 = 4J.$$

4. **(46)** We have, $i = i_0 e^{-tR/L}$

or $10^{-3} = 1 \times e^{-1R/10}$

$\Rightarrow R = 46\,\Omega$

5. **(0.173)** We have, $\tan\theta = \dfrac{B_V}{B_H}$

$\therefore B_V = B_H \tan\theta$

$= 2 \times 10^{-5} \times \tan 60°$

$= 2\sqrt{3} \times 10^{-5}\,\text{T}$

The induced emf, $e = B_v v\ell$

$= (2\sqrt{3} \times 10^{-5}) \times 250 \times 20$

$= 0.173\,\text{V}$

6. **(437.5)** According to faraday's law of electromagnetic induction, $e = \dfrac{-d\phi}{dt}$

$$L \times \frac{di}{dt} = 25 \Rightarrow L \times \frac{15}{1} = 25$$

or, $L = \dfrac{5}{3}\,\text{H}$

Change in the energy of the inductance,

$$\Delta E = \frac{1}{2} L\left(i_1^2 - i_2^2\right) = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2)$$

$$= \frac{5}{6} \times 525 = 437.5\,\text{J}$$

7. **(1.5×10^{-3})** Induced emf, $\varepsilon = Bv\ell$

$= 0.3 \times 10^{-4} \times 5 \times 10$

$= 1.5 \times 10^{-3}\,\text{V}$

8. **(9.3×10^{-2})** The resistance of the wire

$$R = \frac{\rho\ell}{\pi r^2} = \frac{(1.7 \times 10^{-8}) \times (0.40)}{\pi(10^{-3})^2}$$

$= 2.16 \times 10^{-3}\,\Omega$

The area of the loop $= 0.10 \times 0.10 = 10^{-2}\,\text{m}^2$.

The induced emf, $|e| = A\left(\dfrac{dB}{dt}\right)$

$= 10^{-2} \times (0.02) = 2 \times 10^{-4}\,\text{V}$

The induced current, $i = \dfrac{e}{R} = \dfrac{2 \times 10^{-4}}{2.16 \times 10^{-3}}$

$= 9.3 \times 10^{-2}\,\text{A}$.

9. **(7.85×10^{-2})** The magnetic flux, $\phi_B = BA = B \times \pi r^2$

The induced emf, $|e| = \frac{\Delta\phi_B}{\Delta t} = \frac{B \times \pi r^2}{\Delta t}$

$= \frac{0.50 \times \pi(0.05)^2}{0.50}$

$= 7.85 \times 10^{-2}$ V

10. **(0.36)** The required ratio, $\frac{E_{\text{stored}}}{E_{\text{Supplied}}} = \frac{V^2/R}{V_0^2/R}$

$= \frac{V^2}{V_0^2} = \frac{V_0^2(1-e^{-t/\tau})}{V_0^2}$

$= (1 - e^{-t/\tau})^2$

$= \left[1 - e^{-\left(\frac{0.1\times10}{1}\right)}\right]^2$

$= 0.36$

11. **(0.048)** $B = \mu_0 n i = (4\pi \times 10^{-7})(200 \times 10^{-2}) \times 1.5$

$= 3.8 \times 10^{-2}$ Wb / m^2

Magnetic flux through each turn of the coil

$\phi = BA = (3.8 \times 10^{-2})(3.14 \times 10^{-4}) = 1.2 \times 10^{-5}$ weber

When the current in the solenoid is reversed, the change in magnetic flux

$= 2 \times (1.2 \times 10^{-5}) = 2.4 \times 10^{-5}$ weber

Induced e.m.f. $= N\frac{d\phi}{dt} = 100 \times \frac{2.4 \times 10^{-5}}{0.05} = 0.048$ V.

12. **(0.02)** Here, $B = \frac{1}{p}$ (Wb/m^2)

$\theta = 60°$

Area normal to the plane of the disc

$= pr^2 \cos 60° = \frac{pr^2}{2}$

Flux = B × normal area

$= \frac{0.2 ´ 0.2}{2} = 0.02$ Wb

13. **(3)** ϕ (flux linked) $= a^2 B \cos 0° - b^2 B \cos 180°$

$E = -\frac{d\phi}{dt} = -(a^2 - b^2)\frac{dB}{dt} = (a^2 - b^2) B_0 \omega \cos \omega t$

where $B = B_0 \sin \omega t$, $B_0 = 10^{-3}$ T, $\omega = 100$

$\therefore \quad I_{max} = (a^2 - b^2)\dfrac{B_0\omega}{R}$

and $R = (4a + 4b)\,r = 4\,(a + b)\,r$

$\therefore \quad I_{max} = \dfrac{(a-b)B_0\omega}{4r} = \dfrac{(1-0.4)\times 10^{-3}\times 100}{4\times 5\times 10^{-3}} = 3A$

14. **(2)** $\varepsilon = B\ell v = 2\times 10^{-1}\times v = 0.2\ v$

$I = \dfrac{\varepsilon}{R} = 10^{-3} \Rightarrow \dfrac{0.2v}{4} = 10^{-3}$

[Since effective resistance R of bridge is

$R = \dfrac{6\times 6}{6+6} = 3\Omega$ so total resistance $= 1 + 3 = 4\Omega$]

$\Rightarrow v = 2\ \text{cm s}^{-1}$

15. **(9×10^{-3})** Power $= \dfrac{B^2v^2l^2}{R}$

$= \dfrac{0.5\times 0.5\times 12\times 12\times 15\times 15\times 10^{-8}}{9\times 10^{-3}} = 9\times 10^{-3}$ watt